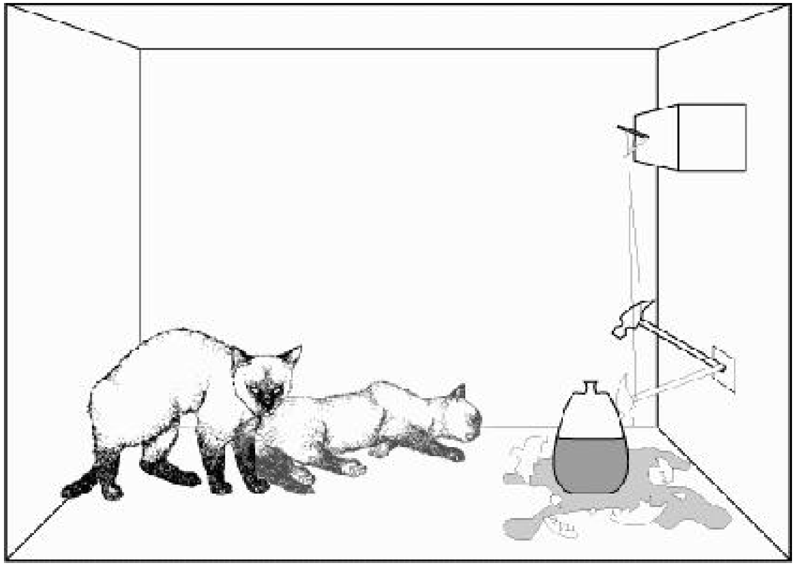
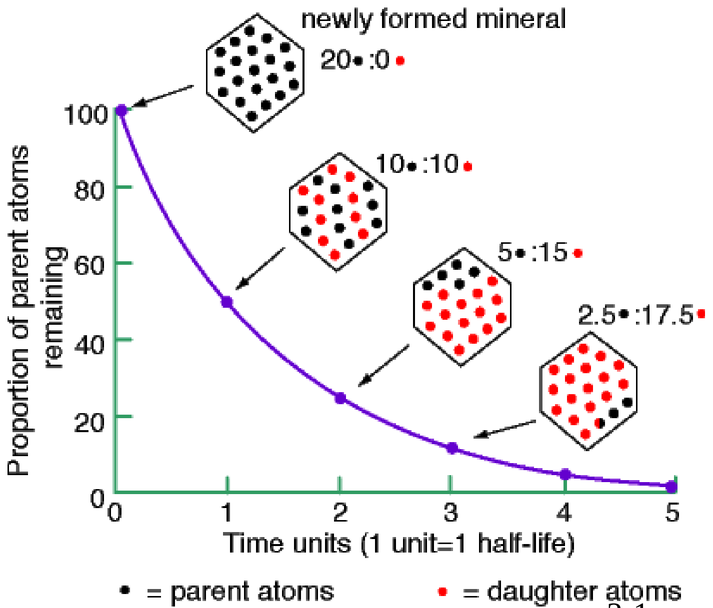
CHEM 702: Lecture 6
Radioactive Decay Kinetics

Outline

- Readings: Modern Nuclear Chemistry Chapter 3; Nuclear and Radiochemistry Chapters 4 and 5
- Radioactive decay kinetics
  - Basic decay equations
  - Utilization of equations
    - → Mixtures
    - → Equilibrium
    - → Branching
    - → Cross section
  - Natural radiation
  - Dating





Introduction to Radioactive Decay

- Number of radioactive nuclei that decay in radioactive sample decreases with time
  - Exponential decrease
  - Independent of P, T, mass action and 1st order
    - → Conditions associated with chemical kinetics
      - * Electron capture and internal conversion can be affected by conditions
  - Specific for isotope and irreversible
- Decay of given radionuclide is random
- Decay rate proportional to amount of parent isotope
  - rate of decay=decay constant*# radioactive nuclei
    - * A= λN
- Decay constant is average decay probability per nucleus for a unit time
- Represented by λ

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Basic decay equations

- **Probability of disintegration for a given radioactive atom in a specific time interval is independent past history and present circumstances**
  - **Probability of disintegration depends only on length of time interval**
- **Probability of decay: $p=\lambda\Delta t$**
- **Probability of not decaying: $1-p=1-\lambda\Delta t$**
  - **$(1-\lambda\Delta t)^n$=probability that atom will survive n intervals of Δt**
  - **$n\Delta t=t$, therefore $(1-\lambda\Delta t)^n=(1-\lambda t/n)^n$**
- **$\lim_{n\to\infty}(1+x/n)^n=e^x$, $(1-\lambda t/n)^n=e^{-\lambda t}$ is limiting value**
- **Considering N_o initial atoms**
  - **fraction remaining unchanged after time t is**

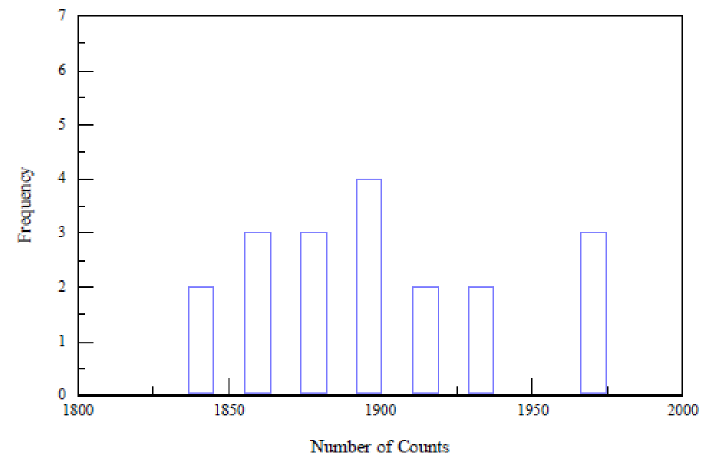
    → $N/N_o= e^{-\lambda t}$

    * **N is number of atoms remaining at time t**

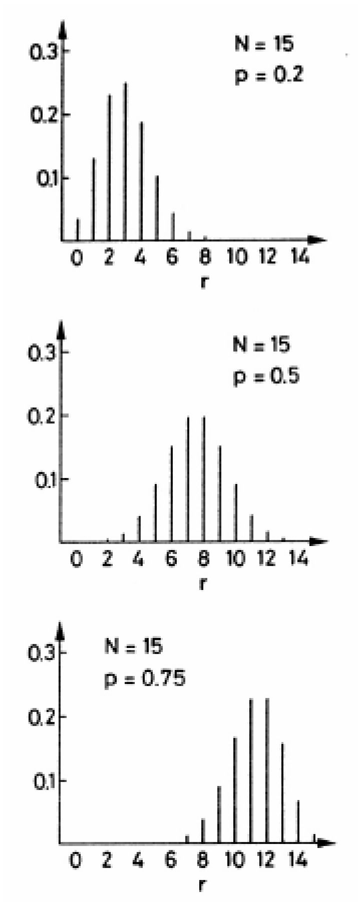
$$N=N_o e^{-\lambda t}$$

Radioactivity as Statistical Phenomenon: Binomial Distribution

- **Radioactive decay a random process**
  - **Number of atoms in a given sample that will decay in a given Δt can differ**
    - → **Neglecting same Δt over large time differences, where the time difference is on the order of a half life**
    - → **Relatively small Δt in close time proximity**
- **Binomial Distribution for Radioactive Disintegrations**
  - **Reasonable model to describe decay process**
    - → **Bin counts, measure number of occurrences counts fall in bin number**
    - → **Can be used as a basis to model radioactive case**
    - → **Classic description of binomial distribution by coin flip**
- **Probability P(x) of obtaining x disintegrations in bin during time t, with t short compared to $t_{1/2}$**
  - **n: number of trials**
  - **p: probability of event in bin**



$$P(x) = \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x}$$



Radioactivity as Statistical Phenomenon: Error from Counting

- **For radioactive disintegration**
  - **Probability of atom not decaying in time t, 1-p, is $(N/N_o)=e^{-\lambda t}$**

    $$P(x) = \frac{N_o!}{(N_o - x)!\,x!}(1-e^{-\lambda t})^x (e^{-\lambda t})^{N_o - x}$$

    →$p=1-e^{-\lambda t}$

    →N is number of atoms that survive in time interval t and N_o is initial number of atoms
- **Time Intervals between Disintegrations**
  - **Distribution of time intervals between disintegrations**

    →t and t+d

    * Write as P(t)dt

    $$P(t)dt = N_o \lambda e^{-N_o \lambda t} dt$$

Decay Statistics

- **Average disintegration rate**
  - **Average value for a set of numbers that obey binomial distribution**
  - **Use n rather than N_o, replace x (probability) with r (disintegrations)**

$$P(x) = \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x} \longrightarrow p(r) = \frac{n!}{(n-r)!\,r!} p^r (1-p)^{n-r}$$

- **Average value for r**

$$\bar{r} = \sum_{r=0}^{r=n} r\,p(r) = \sum_{r=0}^{r=n} r \frac{n!}{(n-r)!\,r!} p^r (1-p)^{n-r}$$

- **Solve using binomial expansion**

$$(px + (1-p))^n = \sum_{r=0}^{r=n} \frac{n!}{(n-r)!\,r!} p^r x^r (1-p)^{n-r} = \sum_{r=0}^{r=n} x^r p(r)$$

  - **Then differentiate with respect to x**

$$np(px + (1-p))^{n-1} = \sum_{r=0}^{r=n} r x^{r-1} p(r)$$

Decay Statistics

- **Let x=1**

$$np(px+(1-p))^{n-1}=\sum_{r=0}^{r=n} rx^{r-1}p(r) \Longrightarrow np=\sum_{r=0}^{r=n} rp(r)=\bar{r}$$

  - **Related to number and probability**
- **For radioactive decay n is N_o and p is (1-$e^{-\lambda t}$)**
- **Use average number of atoms disintegrating in time t**
  - **M=average number of atoms disintegrating in time t**
    - → **Can be measured as counts on detector**
  - **$M=N_o(1-e^{-\lambda t})$**
  - **For small λt, $M=N_o\lambda t$**
  - **Disintegration rate is M per unit time**
    - → **$R=M/t=N_o\lambda$**
    - → **Small λt means count time is short compared to half life**
    - → **Corresponds to -dN/dt=λN=A**

Decay Statistics

- **Expected Standard Deviation**
  - **Base on expected standard deviation from binomial distribution**
  - **Use binomial expansion** $np(px+(1-p))^{n-1}=\sum_{r=0}^{r=n} r x^{r-1} p(r)$
  - **and differentiate with respect to x**

$$n(n-1)p^2(px+(1-p))^{n-2}=\sum_{r=0}^{r=n} r(r-1)x^{r-2}p(r)$$

From bottom of slide 3-6

- **x=1 and p+(p-1)= 1**

$$n(n-1)p^2=\sum_{r=0}^{r=n} r(r-1)p(r)=\sum_{r=0}^{r=n} r^2 p(r)-\sum_{r=0}^{r=n} r p(r)$$

$$n(n-1)p^2=\overline{r^2}-\overline{r}$$

- **Variation defined as** $\sigma_r^2=\overline{r^2}-\overline{r}^2$
- **Combine** $\sigma_r^2=n(n-1)p^2+\overline{r}-\overline{r}^2$

Expected Standard Deviation

- **Solve with:** $\bar{r} = np$

$$\sigma_r^2 = n^2p^2 - np^2 + np - n^2p^2 = np(1-p)$$

$$\sigma_r = \sqrt{np(1-p)}$$

- **Apply to radioactive decay**
  - **M is the number of atoms decaying**

    →Number of counts for a detector

$$\sigma = \sqrt{N_o(1-e^{-\lambda t})e^{-\lambda t}} = \sqrt{Me^{-\lambda t}}$$

$$Since\ in\ counting\ practice\ \lambda t\ is\ small, e^{-\lambda t} \approx 1$$

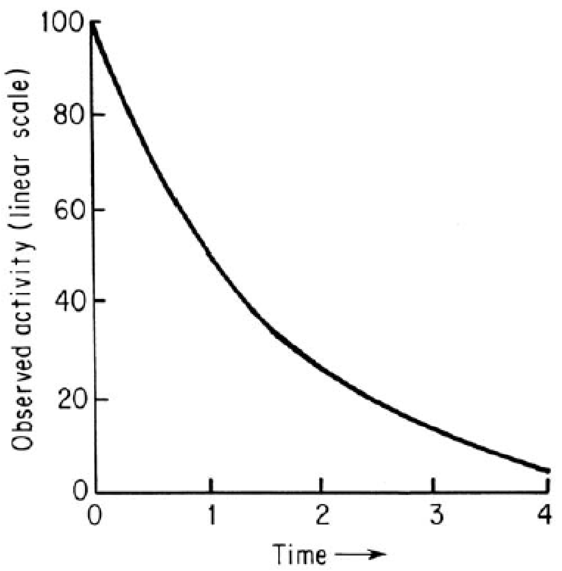
$$\sigma = \sqrt{M}$$

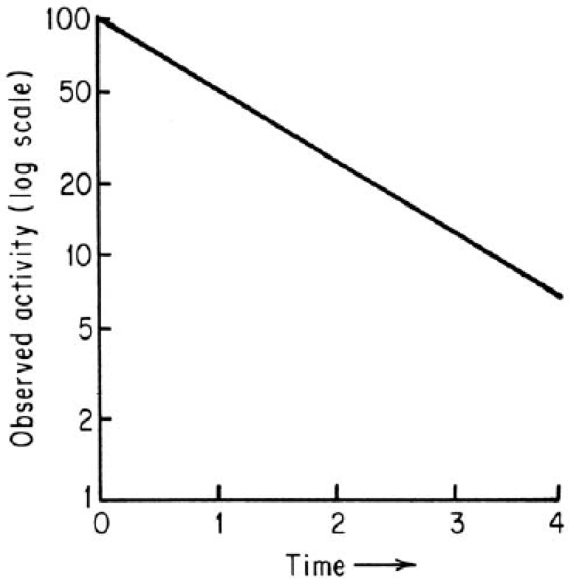
- **Relative error = σ^{-1}**
- **<u>What is a reasonable number of counts</u>**
  - **More counts, lower error**

Counts	error	% error
10	3.16	31.62
100	10.00	10.00
1000	31.62	3.16
10000	100.00	1.00

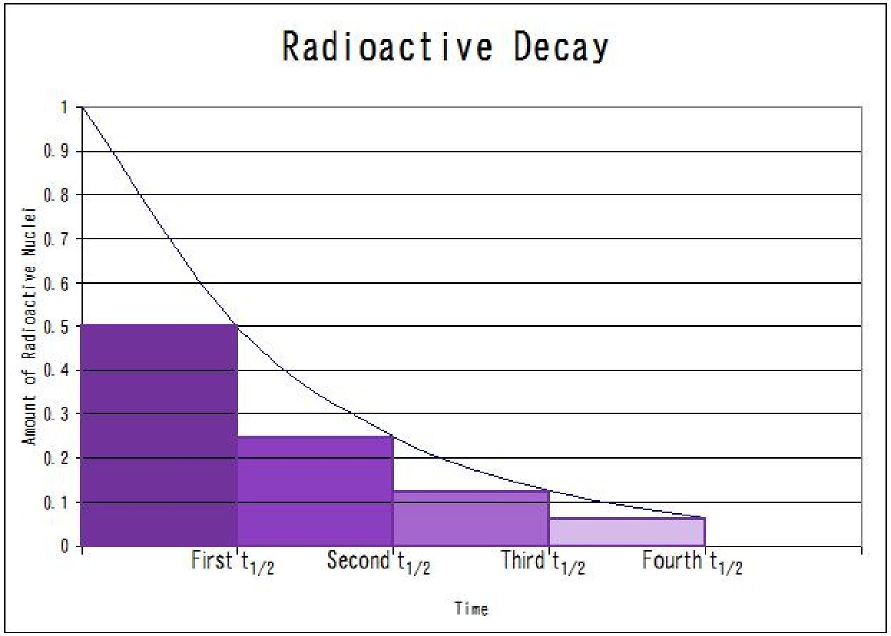
Measured Activity

- Activity (A) determined from measured counts by correcting for geometry and efficiency of detector
  - Not every decay is observed
  - Convert counts to decay
- $A = \lambda N$
- $A = A_o e^{-\lambda t}$
- Units
  - Curie
  - 3.7E10 decay/s
    - → 1 g ^{226}Ra
      - * $A = \lambda N$
- Becquerel
  - 1 decay/s





Half Life and Decay Constant



- **Half-life is time needed to decrease nuclides by 50%**
- **Relationship between $t_{1/2}$ and λ**
- $N/N_o=1/2=e^{-\lambda t}$
- $\ln(1/2)=-\lambda t_{1/2}$
- $\ln 2= \lambda t_{1/2}$
- $t_{1/2}=(\ln 2)/\lambda$

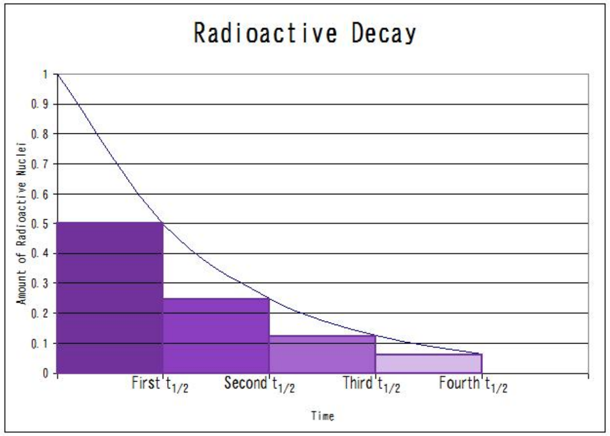
- **Large variation in half-lives for different isotopes**
  - **Short half-lives can be measured**
    - → **Evaluate activity over time**
      - * **Observation on order of half-life**
  - **Long half-lives**
    - → **Based on decay rate and sample**
      - * **Need to know total amount of nuclide in sample**
      - * **$A=\lambda N$**

Exponential Decay

- Average Life (τ) for a radionuclide
  - found from sum of times of existence of all atoms divided by initial number of nuclei

$$\tau = -\frac{1}{N_o}\int_{t=0}^{t=\infty} t \cdot dN = \frac{1}{\lambda}$$

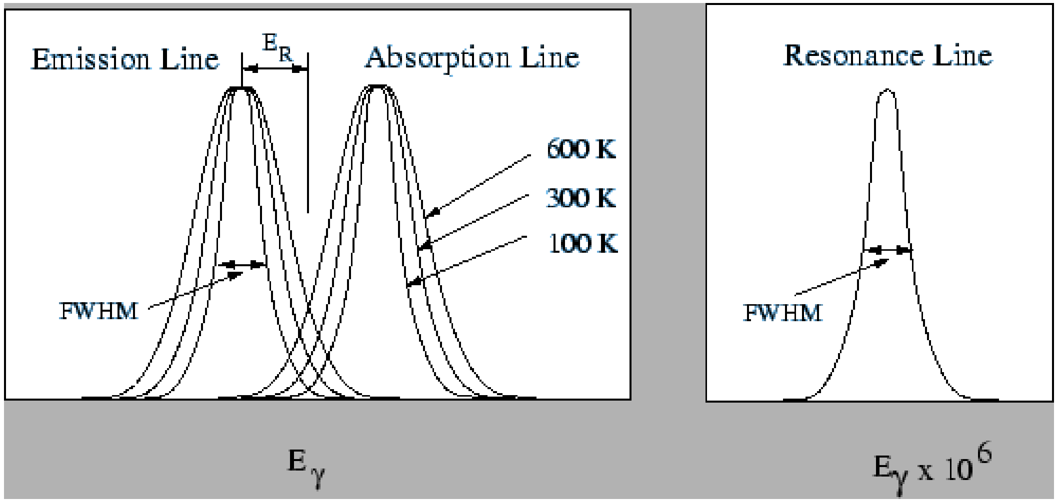
  - $1/\lambda = 1/(\ln 2/t_{1/2}) = 1.443 t_{1/2} = \tau$
    - →Average life greater than half life by factor of 1/0.693
    - →During time 1/λ activity reduced to 1/e it's initial value



- Total number of nuclei that decay over time
  - Dose
  - Atom at a time

Gamma decay and Mossbauer

- Couple with Heisenberg uncertainty principle
- $\Delta E \, \Delta t \geq h/2\pi$
  - Δt is τ, with energy in eV
- $\Delta E \geq (4.133E\text{-}15 \text{ eV s}/2\pi)/\tau = \Gamma$
- Γ is decay width
  - Resonance energy
  - $\Gamma(\text{eV}) = 4.56E\text{-}16/t_{1/2}$ seconds
    - → $t_{1/2}$=1 sec, τ=1.44 s
- Need very short half-lives for large widths
- Useful in Moessbauer spectroscopy
  - Absorption distribution is centered around $E_\gamma + \Delta E$
  - emission centered $E_\gamma - \Delta E$.
- overlapping part of the peaks can be changed by changing temperature of source and/or absorber
- Doppler effect and decay width result in energy distribution near E_r
  - Doppler from vibration of source or sample



Important Equations!

- $N_t = N_o e^{-\lambda t}$
  - N=number of nuclei, λ= decay constant, t=time

    → Also works for A (activity) or C (counts)

    * $A_t = A_o e^{-\lambda t}$, $C_t = C_o e^{-\lambda t}$
- $A = \lambda N$
- $1/\lambda = 1/(\ln 2/t_{1/2}) = 1.443 t_{1/2} = \tau$
- Error
  - M is number of counts $\sigma = \sqrt{M}$

Half-life calculation

Using $N_t=N_o e^{-\lambda t}$

- **For an isotope the initial count rate was 890 Bq. After 180 minutes the count rate was found to be 750 Bq**
  - **What is the half-life of the isotope**
    - →$750=890\exp(-\lambda*180\text{ min})$
    - →$750/890=\exp(-\lambda*180\text{ min})$
    - →$\ln(750/890)= -\lambda*180\text{ min}$
    - →$-0.171/180\text{ min}= -\lambda$
    - →$9.5\text{E}–4\text{ min}^{-1} =\lambda=\ln2/t_{1/2}$
    - →$t_{1/2}=\ln2/9.5\text{E-}4=729.6\text{ min}$

Half-life calculation

$A=\lambda N$

- **A 0.150 g sample of ^{248}Cm has a alpha activity of 0.636 mCi.**
  - **What is the half-life of ^{248}Cm?**
    - → **Find A**
      - * **0.636 E-3 Ci (3.7E10 Bq/Ci)=2.35E7 Bq**
    - → **Find N**
      - * **0.150 g x 1 mole/248 g x 6.02E23/mole= 3.64E20 atoms**
    - → **λ=A/N= 2.35E7 Bq/3.64E20 atoms=6.46E-14 s^{-1}**
      - * **$t_{1/2}=\ln 2/\lambda=0.693/6.46E-14\ s^{-1}=1.07E13\ s$**
      - * **1.07E13 s=1.79E11 min=2.99E9 h=1.24E8 d =3.4E5 a**

Counting

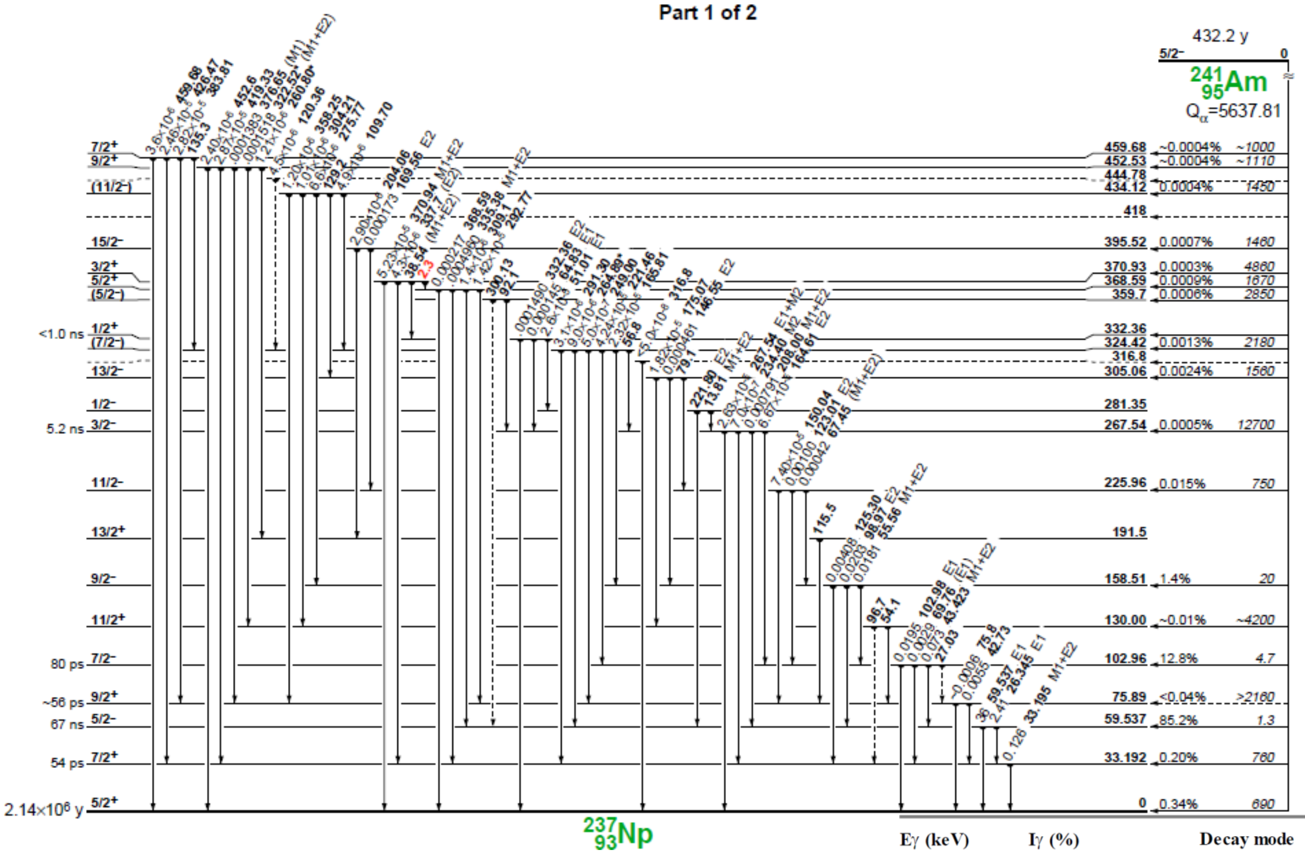
$A=\lambda N$

- **Your gamma detector efficiency at 59 keV is 15.5 %. What is the expected gamma counts from 75 micromole of ^{241}Am?**
    - **Gamma branch is 35.9 % for ^{241}Am**
    - **$C=(0.155)(0.359)\lambda N$**
    - **$t_{1/2}$=432.7 a* (3.16E7 s/a)=1.37E10 s**
    - **λ=ln2/1.37E10 s=5.08E-11 s^{-1}**
    - **N=75E-6 moles *6.02E23/mole=4.52E19 atoms**
- **C=(0.155)(0.359)5.08E-11 s^{-1}*4.52E19 =1.28E8 counts/second**

$\gamma(^{237}Np)$ from ^{241}Am (432.2 y) α decay < *for Iγ% multiply by 1.00×10^{-5} 1>*

13.81 *2*
26.345 *1* ($\dagger_\gamma$2.41×10^5 *5*) E1
27.03 (?)
31.4
32.183 (u) ($\dagger_\gamma$1740 *180*)
33.205 *10* ($\dagger_\gamma$12600 *100*) M1+E2: δ=0.13 *3*
38.54 *3*
42.73 *5* ($\dagger_\gamma$ 550 *110*) (M1+E2): δ≈0.86
43.423 *10* ($\dagger_\gamma$7300 *800*) M1+E2: δ=0.41 *2*
51.01 *3* ($\dagger_\gamma$2.6 *12*)
54.1
55.56 *2* ($\dagger_\gamma$1810 *180*) M1+E2: δ=0.46 *4*
56.8
57.85 *5* (u) ($\dagger_\gamma$ 520 *150*)
59.537 *1* ($\dagger_\gamma$3.59×10^6) E1

Decay Scheme



$\gamma(^{237}Np)$ from ^{241}Am (432.2 y) α decay < *for Iγ% multiply by 1.00×10*$^{-5}$ *1*>

13.81 *2*

26.345 *1* ($\dagger_\gamma$ 2.41×10^5 *5*) E1

27.03 (?)

31.4

32.183 (u) ($\dagger_\gamma$ 1740 *180*)

33.205 *10* ($\dagger_\gamma$ 12600 *100*) M1+E2: δ=0.13 *3*

38.54 *3*

42.73 *5* ($\dagger_\gamma$ 550 *110*) (M1+E2): δ≈0.86

43.423 *10* ($\dagger_\gamma$ 7300 *800*) M1+E2: δ=0.41 *2*

51.01 *3* ($\dagger_\gamma$ 2.6 *12*)

54.1

55.56 *2* ($\dagger_\gamma$ 1810 *180*) M1+E2: δ=0.46 *4*

56.8

57.85 *5* (u) ($\dagger_\gamma$ 520 *150*)

59.537 *1* ($\dagger_\gamma$ 3.59×10^6) E1

Eγ (keV)	Iγ (%)	Decay mode	Half life	Parent
13.81 *2*		α	432.2 y *7*	^{241}Am
26.3448 *2*	2.40 *2*	α	432.2 y *7*	^{241}Am
27.03		α	432.2 y *7*	^{241}Am
31.4		α	432.2 y *7*	^{241}Am
32.183	0.0174 *4*	α	432.2 y *7*	^{241}Am
33.1964 *3*	0.126 *3*	α	432.2 y *7*	^{241}Am
38.54 *3*		α	432.2 y *7*	^{241}Am
42.73 *5*	0.0055 *11*	α	432.2 y *7*	^{241}Am
43.423 *10*	0.073 *8*	α	432.2 y *7*	^{241}Am
51.01 *3*	0.000026 *12*	α	432.2 y *7*	^{241}Am
54.0		α	432.2 y *7*	^{241}Am
55.56 *2*	0.0181 *18*	α	432.2 y *7*	^{241}Am
56.8		α	432.2 y *7*	^{241}Am
57.85 *5*	0.0052 *15*	α	432.2 y *7*	^{241}Am
59.5412 *2*	35.9 *4*	α	432.2 y *7*	^{241}Am

Specific activity

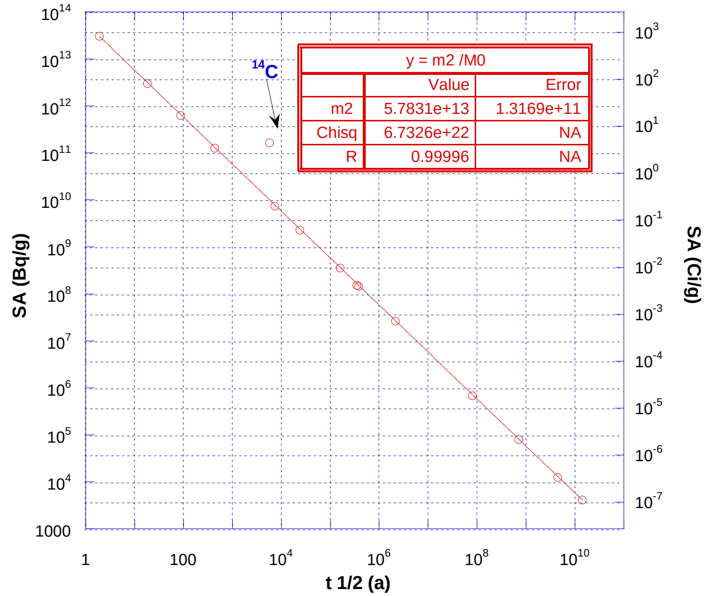
- **Activity of a given amount of radionuclide**
  - **Use $A=\lambda N$**

    →Use of carrier should be included
- **SA of ^{226}Ra**
  - **1 g ^{226}Ra, $t_{1/2}$= 1599 a**
  - **1 g * 1 mole/226 g * 6.02E23 atoms/mole = 2.66E21 atom = N**
  - **$t_{1/2}$=1599 a *3.16E7 s/a = 5.05E10 s**

    →λ=ln2/ 5.05E10 s =1.37E-11 s^{-1}
  - **A= 1.37E-11 s^{-1} * 2.66E21=3.7E10 Bq**
  - **Definition of a Curie!**

Specific Activity

- 1 g ^{244}Cm, $t_{1/2}$=18.1 a
  - 1 g * 1 mole/244 g * 6.02E23 atoms/mole = 2.47E21 atom = N
  - $t_{1/2}$=18.1 a *3.16E7 s/a = 5.72E8 s
    → λ=ln2/ 5.72E8 s =1.21E-9 s^{-1}
  - A= 1.21E-9 s^{-1} * 2.47E21=2.99E12 Bq
- Generalized equation for 1 g
  - 6.02E23/Isotope mass *2.19E-8/ $t_{1/2}$ (a)
  - 1.32E16/(Isotope mass* $t_{1/2}$ (a))

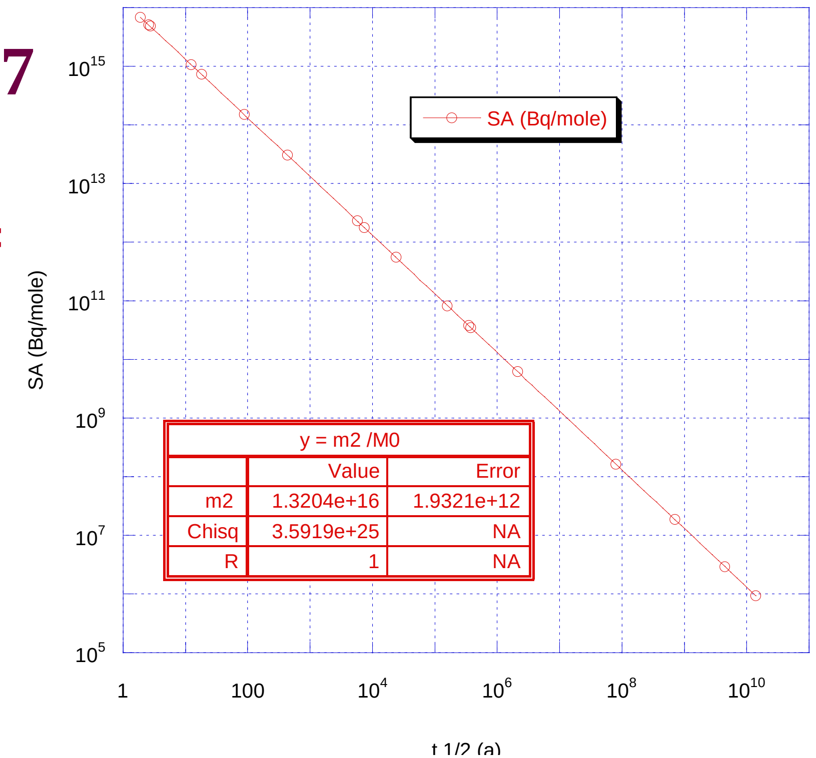


Isotope		$t_{1/2}$ (a)	SA (Bq/g)
14	C	5715	1.65E+11
228	Th	1.91E+00	3.03E+13
232	Th	1.40E+10	4.06E+03
233	U	1.59E+05	3.56E+08
235	U	7.04E+08	7.98E+04
238	U	4.47E+09	1.24E+04
237	Np	2.14E+06	2.60E+07
238	Pu	8.77E+01	6.32E+11
239	Pu	2.40E+04	2.30E+09
242	Pu	3.75E+05	1.45E+08
244	Pu	8.00E+07	6.76E+05
241	Am	4.33E+02	1.27E+11
243	Am	7.37E+03	7.37E+09
244	Cm	1.81E+01	2.99E+12
248	Cm	3.48E+05	1.53E+08

Specific Activity

- **Activity/mole**
  - **N=6.02E23**
- **SA (Bq/mole) of ^{129}I, $t_{1/2}$=1.57E7 a**
  - **$t_{1/2}$=1.57E7 a *3.16E7 s/a = 4.96E14 s**

    →λ=ln2/ 4.96E14 s =1.397E-15 s^{-1}
  - **A= 1.397E-15 s^{-1} *6.02E23=8.41E8 Bq**
- **Generalized equation**
  - **SA (Bq/mole)=1.32E16/$t_{1/2}$ (a)**



Specific activity with carrier

- **1E6 Bq of ^{152}Eu is added to 1 mmole Eu.**
  - **Specific activity of Eu (Bq/g)**
  - **Need to find g Eu**
    - **→1E-3 mole *151.96 g/mole = 1.52E-1 g**
    - **→=1E6 Bq/1.52E-1 g =6.58E6 Bq/g**
      - **=1E9 Bq/mole**
- **What is SA after 5 years**
  - **$t_{1/2}$=13.54 a**
    - **→= 6.58E6*exp((-ln2/13.54)*5)=**
      - **5.09E6 Bq/g**

Lifetime

- **Atom at a time chemistry**
- **^{261}Rf lifetime**
  - **Find the lifetime for an atom of ^{261}Rf**

    → $t_{1/2} = 65$ s

    → $\tau = 1.443 t_{1/2}$

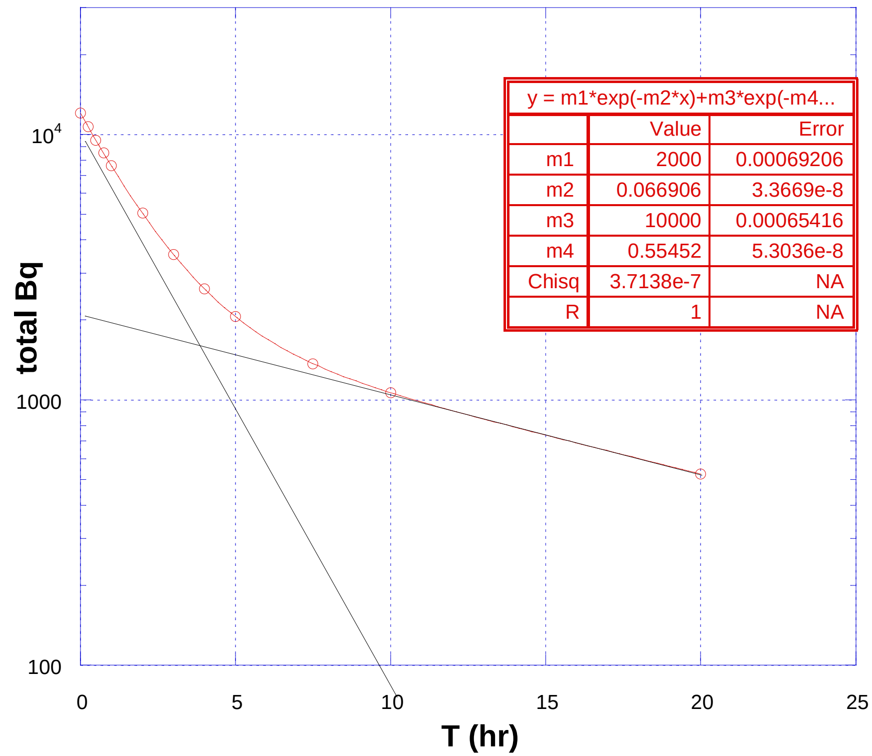
    → $\tau = 93$ s
- **Determines time for experiment**
- **Method for determining half-life**

Mixtures of radionuclides

- Composite decay
  - Sum of all decay particles
    - → Not distinguished by energy
- Mixtures of Independently Decaying Activities
  - if two radioactive species mixed together, observed total activity is sum of two separate activities:

    $$A_t = A_1 + A_2 = \lambda_1 N_1 + \lambda_2 N_2$$

  - any complex decay curve may be analyzed into its components
    - → Graphic analysis of data is possible



$\lambda = 0.554\ hr^{-1}$

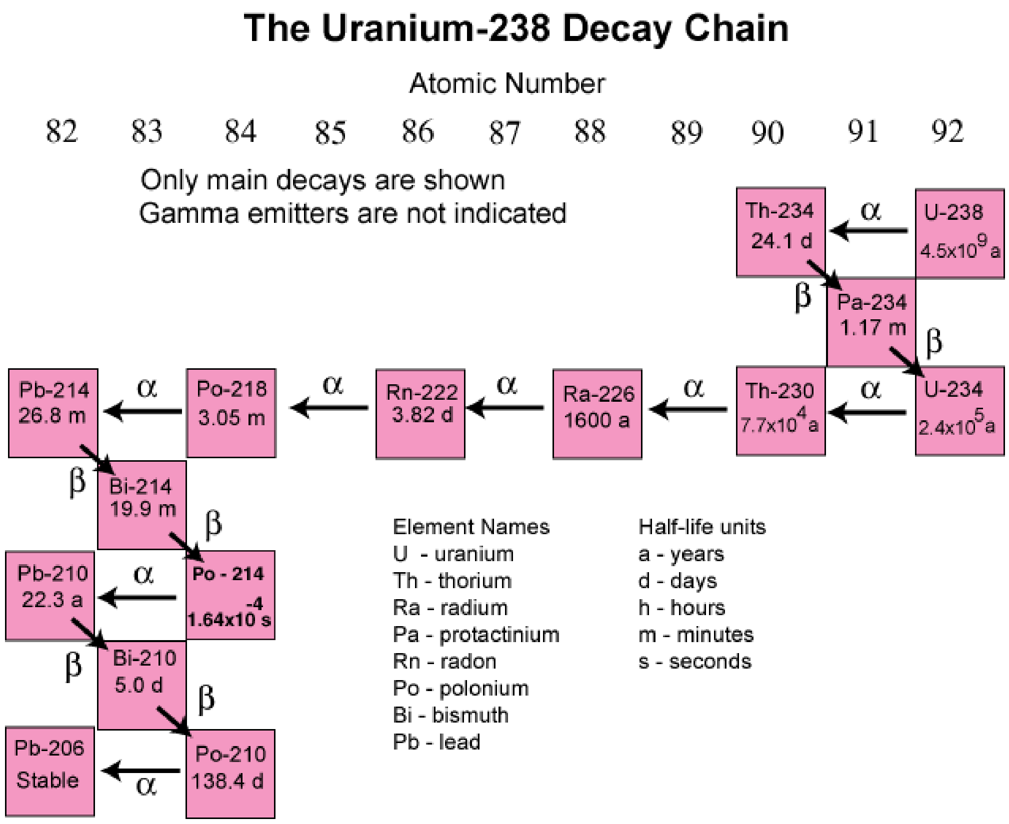
$t_{1/2} = 1.25\ hr$

$\lambda = 0.067\ hr^{-1}$

$t_{1/2} = 10.4\ hr$

Parent – daughter decay

- Isotope can decay into radioactive isotope
  - Uranium and thorium decay series
    - → Alpha and beta
      - * A change from alpha decay
- Different designation
  - 4n (^{232}Th)
  - 4n+2 (^{238}U)
  - 4n+3 (^{235}U)
- For a decay parent -> daughter
  - Rate of daughter formation dependent upon parent decay rate-daughter decay rate



Parent - daughter

- **How does daughter isotope change with parent decay**
  - **isotope 1 (parent) decays into isotope 2 (daughter)**

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

- **Rearranging gives** $dN_2 + \lambda_2 N_2 dt = \lambda_1 N_1 dt$
- **Solve and substitute for N_1 using $N_{1t}=N_{1o}e^{-\lambda t}$**

$$dN_2 + \lambda_2 N_2 dt = \lambda_1 N_{1o} e^{-\lambda_1 t} dt$$

  - **Linear 1^{st} order differential equation**

    →**Solve by integrating factors**

- **Multiply by $e^{\lambda 2t}$**

$$e^{\lambda_2 t} dN_2 + \lambda_2 N_2 e^{\lambda_2 t} dt = \lambda_1 N_{1o} e^{(\lambda_2 - \lambda_1)t} dt$$

$$d(N_2 e^{\lambda_2 t}) = \lambda_1 N_{1o} e^{(\lambda_2 - \lambda_1)t} dt$$

Parent-daughter

- **Integrate over t**

$$\int_0^t N_2 e^{\lambda_2 t} = \int_0^t \frac{\lambda_1 N_{1o} e^{(\lambda_2 - \lambda_1)t}}{\lambda_2 - \lambda_1}$$

$$N_2 e^{\lambda_2 t} - N_{2o} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1o} (e^{(\lambda_2 - \lambda_1)t} - 1)$$

- **Multiply by $e^{-\lambda_2 t}$ and solve for N_2**

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1o} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_{2o} e^{-\lambda_2 t}$$

Growth of daughter from parent

Initial daughter

Parent daughter relationship

- **Find N, can solve equation for activity from A=λN**

$$A_2 = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_{1o}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_{2o} e^{-\lambda_2 t}$$

- **Find maximum daughter activity based on dN/dt=0**
- **Solve for t**

$$t = \frac{\ln(\frac{\lambda_2}{\lambda_1})}{(\lambda_2 - \lambda_1)}$$

$$\lambda_1 e^{-\lambda_1 t} = \lambda_2 e^{-\lambda_2 t}$$

- **For ^{99m}Tc ($t_{1/2}$=6.01 h) from ^{99}Mo (2.75 d), find time for maximum daughter activity**
  - **λ_{Tc}=2.8 d^{-1}, λ_{Mo}=0.25 d^{-1}**

$$t = \frac{\ln(\frac{2.8}{0.25})}{(2.8 - 0.25)} = \frac{\ln(11.2)}{2.55} = 0.95\,days$$

Half life relationships

- Can simplify relative activities based on half life relationships
- No daughter decay
  - No activity from daughter
  - Number of daughter atoms due to parent decay

$$N_2 = N_{1o}(1 - e^{-\lambda_1 t})$$

Daughter Radioactive

- No Equilibrium
  - If parent is shorter-lived than daughter ($\lambda_1 > \lambda_2$)
    → no equilibrium attained at any time
  - Daughter reaches maximum activity when $\lambda_1 N_1 = \lambda_2 N_2$
    → All parents decay, then decay is based on daughter

Half life relationships

- **Transient equilibrium**
  - **Parent half life greater than 10 x daughter half life**

    →($\lambda_1 < \lambda_2$)
- **Parent daughter ratio becomes constant over time**
  - **As t goes toward infinity**

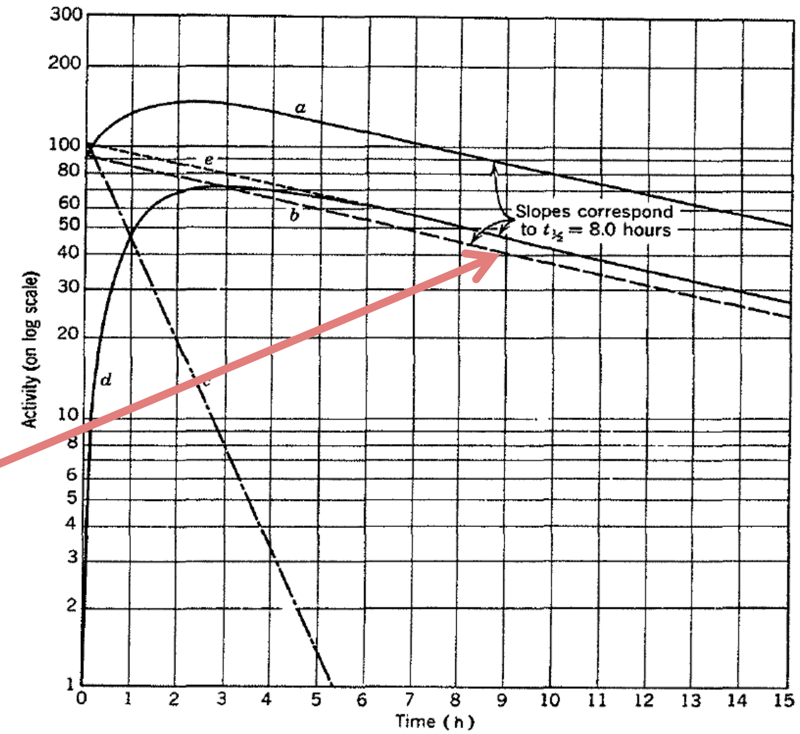


Fig. 5-2 Transient equilibrium: (*a*) total activity of an initially pure parent fraction; (*b*) activity due to parent ($t_{1/2}$ = 8.0 h); (*c*) decay of freshly isolated daughter fraction ($t_{1/2}$ = 0.80 h); (*d*) daughter activity growing in freshly purified parent fraction; (*e*) total daughter activity in parent-plus-daughter fractions.

$$e^{-\lambda_2 t} << e^{-\lambda_1 t};\, N_{2o} e^{-\lambda_2 t} \longrightarrow 0$$

$$N_2 \approx \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1o} e^{-\lambda_1 t}$$

$$\mathbf{N}_1 = \mathbf{N}_{1o} \mathbf{e}^{-\lambda_1 \mathbf{t}}$$

$$\frac{\mathbf{N}_2}{\mathbf{N}_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

Half life relationship

- **Secular equilibrium**
  - **Parent much longer half-life than daughter**

    →1E4 times greater

  ($\lambda_1 << \lambda_2$)
  - **Parent activity does not measurably decrease in many daughter half-lives**

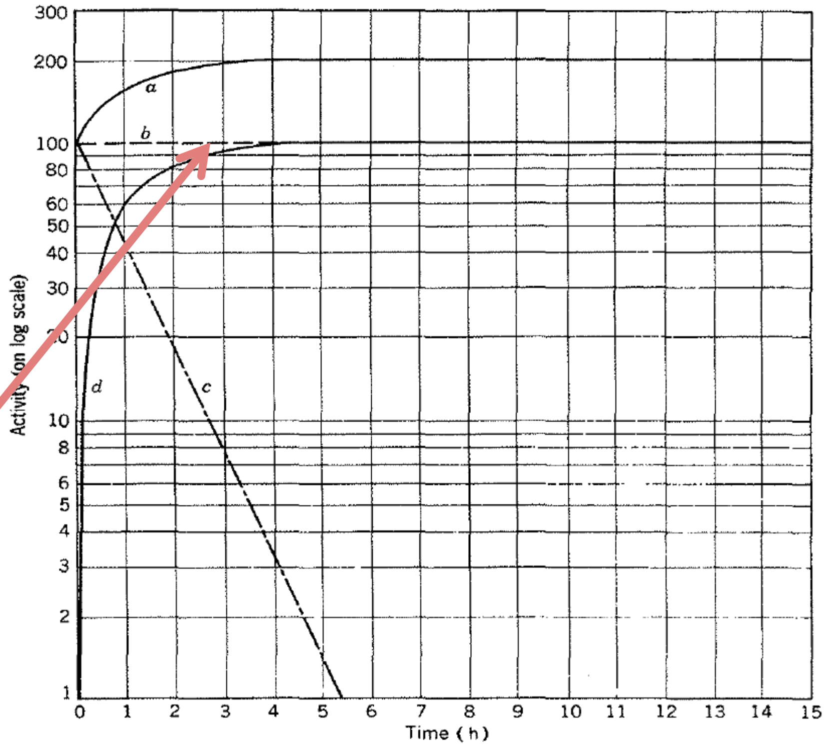


Fig. 5-3 Secular equilibrium: (a) total activity of an initially pure parent fraction; (b) activity due to parent ($t_{1/2} = \infty$); this is also the total daughter activity in parent-plus-daughter fractions; (c) decay of freshly isolated daughter fraction ($t_{1/2} = 0.80$ h); (d) daughter activity growing in freshly purified parent fraction.

$$\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \Rightarrow \frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow$$

$$N_2\lambda_2 = N_1\lambda_1$$

$$A_2 = A_1$$

Many Decays

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3$$

- **Can use the Bateman solution to calculate entire chain**
- **Bateman assumes only parent present at time 0**

$$N_n = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + C_n e^{-\lambda_n t}$$

$$C_1 = \frac{\lambda_1 \lambda_2 \ldots\ldots \lambda_{(n-1)}}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)\ldots(\lambda_n - \lambda_1)} N_{1o}$$

$$C_2 = \frac{\lambda_1 \lambda_2 \ldots\ldots \lambda_{(n-1)}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)\ldots(\lambda_n - \lambda_2)} N_{1o}$$

Program for Bateman http://www.ergoffice.com/downloads.aspx

Review of ERG Program

Environmental radionuclides and dating

- **Primordial nuclides that have survived since time elements were formed**
  - **$t_{1/2}$>1E9 a**
  - **Decay products of these long lived nuclides**

    → ^{40}K, ^{87}Rb, ^{238}U, ^{235}U, ^{232}Th
- **shorter lived nuclides formed continuously by interaction of comic rays with matter**
  - **^{3}H, ^{14}C, ^{7}Be**

    → $^{14}N(n, {}^{1}H)^{14}C$ (slow n)

    → $^{14}N(n, {}^{3}H)^{12}C$ (fast n)
- **anthropogenic nuclides introduced into the environment by activities of man**
  - **Actinides and fission products**
  - **^{14}C and ^{3}H**

Dating

- **Radioactive decay as clock**
  - **Based on $N_t=N_oe^{-\lambda t}$**
    - **→ Solve for t**

$$t = \frac{\ln \frac{N_t}{N_o}}{-\lambda} = \frac{\ln \frac{N_o}{N_t}}{\lambda}$$

- **N_0 and N_t are the number of radionuclides present at times t=0 and t=t**
  - **N_t from $A = \lambda N$**
- ***t* the age of the object**
  - **Need to determine N_o**
    - **→ For decay of parent *P* to daughter *D* total number of nuclei is constant**

$$D(t) + P(t) = P_o$$

Dating

- $P_t = P_o e^{-\lambda t}$

$$t = \frac{1}{\lambda}\ln\left(1+\frac{D_t}{P_t}\right)$$

- Measuring ratio of daughter to parent atoms
  - No daughter atoms present at t=0
  - All daughter due to parent decay
  - No daughter lost during time t
- A mineral has a $^{206}Pb/^{238}U$ =0.4. What is the age of the mineral?

$$t = \frac{1}{\frac{\ln 2}{4.5E9a}}\ln(1+0.4)$$

→2.2E9 years

Dating

$$t = \frac{1}{\lambda}\ln\left(\frac{{}^{14}C_{eq}}{{}^{14}C_{sample}}\right)$$

- **^{14}C dating**
  - **Based on constant formation of ^{14}C**
    - **→No longer uptakes C upon organism death**
- **227 Bq ^{14}C /kgC at equilibrium**
- **What is the age of a wooden sample with 0.15 Bq/g C?**

$$t = \frac{1}{\left(\frac{\ln 2}{5730 years}\right)}\ln\left(\frac{0.227}{0.15}\right) = 3420 years$$

Dating

- **Determine when Oklo reactor operated**
  - **Today 0.7 % ^{235}U**
  - **Reactor 3.5 % ^{235}U**
  - **Compare ^{235}U/^{238}U (U_r) ratios and use $N_t=N_o e^{-\lambda t}$**

$$U_r(t) = U_r(o)\frac{e^{-\lambda_{235}t}}{e^{-\lambda_{238}t}} = U_r(o)e^{(-\lambda_{235}t+\lambda_{238}t)}$$

$$\ln\frac{U_r(t)}{U_r(o)} = t(-\lambda_{235} + \lambda_{238})$$

$$t = \frac{\ln\frac{U_r(t)}{U_r(o)}}{(-\lambda_{235} + \lambda_{238})} \qquad t = \frac{\ln\frac{7.05E-3}{3.63E-2}}{(-9.85E-10 + 1.55E-10)} = 1.97E9\,years$$

Topic review

- **Utilize and understand the basic decay equations**
- **Relate half life to lifetime**
- **Understand relationship between count time and error**
- **Utilization of equations for mixtures, equilibrium and branching**
- **Use cross sections for calculation nuclear reactions and isotope production**
- **Utilize the dating equation for isotope pair**

Study Questions

- **Compare and contrast nuclear decay kinetics and chemical kinetics.**
- **If M is the total number of counts, what is the standard deviation and relative error from the counts?**
- **Define Curie and Becquerel**
- **How can half-life be evaluated?**
- **What is the relationship between the decay constant, the half-life, and the average lifetime?**
- **For an isotope the initial count rate was 890 Bq. After 180 minutes the count rate was found to be 750 Bq. What is the half-life of the isotope?**
- **A 0.150 g sample of ^{248}Cm has a alpha activity of 0.636 mCi. What is the half-life of ^{248}Cm?**
- **What is the half life for each decay mode for the isotope ^{212}Bi?**
- **How are cross sections used to determine isotope production rate?**
- **Determine the amount of ^{60}Co produced from the exposure of 1 g of Co metal to a neutron flux of 10^{14} n/cm^2/sec for 300 seconds.**
- **What are the basic assumptions in using radionuclides for dating?**

Pop Quiz

- **You have a source that is 0.3 Bq and the source is detected with 50 % efficiency. It is counted for 10 minutes. Which total counts shown below are <u>not</u> expected from these conditions?**
- **95, 81, 73, 104, 90, 97, 87**

- **Submit by e-mail or bring to class on 24 September**
- **Comment on Blog**

Useful projects

- **Make excel sheets to calculate**
  - **Mass or mole to activity**
    - **→Calculate specific activity**
  - **Concentration and volume to activity**
    - **→Determine activity for counting**
  - **Isotope production from irradiation**
  - **Parent to progeny**
    - **→Daughter and granddaughter**
      - **i.e., ^{239}U to ^{239}Np to ^{239}Pu**